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LETTER TO THE EDITOR

Dynamics of domain growth in degenerate conserved and non-conserved systems

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Abstract. We numerically study the time-dependent behaviour of domain growth in quenched systems with multiply degenerate ordered states. Based on the phenomenological model equations for such degenerate systems recently proposed by Enomoto *et al*, we perform two-dimensional computer simulations for triply degenerate systems both with a conserved and a non-conserved order parameter, respectively. In particular, we discuss the asymptotic scaling behaviour for the systems and the corresponding growth exponents for the characteristic length scales.

Dynamics of domain growth of quenched systems with multiply degenerate ordered states has attracted increasing attention in many areas of physics [1]. After a quench of degenerate systems such as Cu_3Au and Ni_3Mn alloys [2], clusters of the ordered phases appear within a matrix of a disordered phase. The individual clusters are in any of the p-allowed ordered states, p being the degree of degeneracy for ordered phases. As time goes by, the isolated clusters grow in size and meet each other. The resulting system is then composed of ordered domains in different ordered states, separated by domain walls (called a cellular structure) [2]. These cellular structures can be seen in a wide variety of materials, ranging from metal films to lipid monolayers and magnetic bubbles [1].

To study the coarsening of such cellular structures, two different computational models have been proposed. One is the kinetic p-state Potts model for a conserved [3] and a non-conserved system [4, 5]. The other is the vertex model for a non-conserved system [6, 7]. Performing computer simulations of the above two models, many authors have intensively studied the growth exponent of the average domain size, the scaling behaviour of a domain size distribution function, and their degeneracy dependence. On the other hand, the asymptotic scaling behaviour of the scattering structure factor (SSF) for these degenerate systems has been investigated little in contrast to non-degenerate systems [8, 9].

Recently, we proposed the continuum dynamical model for multiply degenerate systems [10, 11], which can deal with not only the late stage coarsening of cellular structures but also the preceding ordering process of ordered states. In our previous work, we simulated the model equation for multiply degenerate non-conserved systems to visualize the pattern formation of growing domains [10]. We also obtained the time dependence of the average domain size and the scaling behaviour of the domain size distribution function [11], which are similar to those by the above two models. In this letter, we thus focus on the long term behaviour of SSF in multiply degenerate systems both for a conserved and a non-conserved order parameter, respectively. As a first step, we restrict ourselves here to triply degenerate systems.

Our starting equation is the continuum version of the p-state vector Potts model. Let $g(\boldsymbol{r}, t) \equiv F(\boldsymbol{r}, t) \exp(\mathrm{i}S(\boldsymbol{r}, t))$ be the complex scalar field at position $\boldsymbol{r}$ and time t. The amplitude $F(\boldsymbol{r}, t)$ of $g(\boldsymbol{r}, t)$ has a positive value and distinguishes stable ordered states from the disordered ones, while the phase variable $S(\boldsymbol{r}, t)$ describes differences among the multiply degenerate ordered states. We assume that the equation describing the dynamics of the system considered here can be written as [10, 11]:

$$\partial g(\boldsymbol{r}, t)/\partial t = -L(-\mathrm{i}\boldsymbol{\nabla})^{\alpha}(\delta G(g)/\delta g^*) \tag{1}$$

where $\alpha = 2$ corresponds to a conserved system and $\alpha = 0$ to a non-conserved one, respectively, L is a positive constant, and the asterisk denotes the complex conjugate. The coarse-grained free energy functional $G(g)$ is a functional of $g(\boldsymbol{r}, t)$ and is taken to be [10]

$$G(g) \equiv \int \mathrm{d}\boldsymbol{r}(\tfrac{1}{2}|\boldsymbol{\nabla} g|^2 + W(g)) \tag{2}$$

with

$$W(g) \equiv -|g|^2 + \tfrac{1}{4}|g|^4 - \frac{v}{2p}(g^p + g^{*p}) \tag{3}$$

$$= -F^2 + \frac{F^4}{2} - \frac{v}{p}F^p \cos(pS) \tag{4}$$

where p is a positive integer corresponding to the number of degeneracy for ordered states, and v is a positive constant. It is found that if $0 < v < A_p$, $W(g)$ has p-fold degenerate minima at (F_e, S_j), $j = 0, 1, \ldots, p-1$ with

$$-1 + F_e^2 - vF_e^{p-2} = 0 \tag{5}$$

$$S_j = \frac{2\pi}{p}j \tag{6}$$

where A_p is defined by equation (4) in reference [11]. Note that in the present model the thermal noise is neglected and nucleation cannot be dealt with. Note also that the dynamics of a quenched non-conserved system with a continuous symmetry (no degeneracy) has been studied intensively on the basis of a similar model (but $v = 0$ in (3)) [8, 9].

We numerically solve equations (1)–(3) with $p = 3$, using the standard implicit formula on a N^2 square lattice with periodic boundary conditions. We have also set the time step $\Delta t = 0.05$, the lattice spacing $\Delta x = \Delta y = 1$, $v = 0.4$, and $L = 1.0$. In order to solve the dynamical equation for the complex variable, it is convenient to use two real fields, $A(\boldsymbol{r}, t)$ and $B(\boldsymbol{r}, t)$, defined by $g \equiv A + \mathrm{i}B$, rather than the amplitude and phase variable of g [11]. Initially at each lattice site $\boldsymbol{n}$, both real and imaginary parts of the order parameter are chosen to be different Gaussian random numbers with average 0 and variance 0.1, respectively, which effectively represent disordered states.

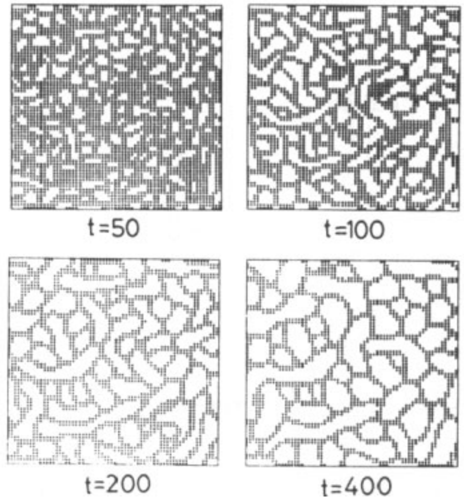

Figure 1. Domain growth in a triply degenerate system with a conserved order parameter. + denotes a disordered lattice site. See the text.

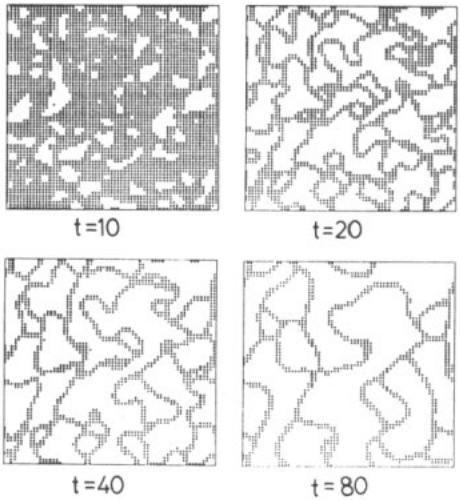

Figure 2. Domain growth in a triply degenerate system with a non-conserved order parameter. + denotes a disordered lattice site. See the text.

In figures 1 and 2 we show the time evolution of the systems with $N = 64$ for a conserved and a non-conserved case, respectively. In these figures, the disordered lattice site $\boldsymbol{n}$ is marked by +, where the disordered site is characterized by the site with $F(\boldsymbol{n}, t)/F_e < 0.9$ or $|S(\boldsymbol{n}, t) - S_j| > \pi/12, j = 0, 1, 2$. From these figures, we can see the emergence and growth of ordered clusters and the subsequent coarsening process among ordered domains. Note that as was pointed out in reference [3], the average domain size of the conserved system seems to be smaller than that of the non-conserved one.

In the following, in order to discuss the asymptotic properties of quenched multiply degenerate systems, we simulate the model equation using a 256^2 square lattice. Moreover, the following results are obtained by averaging over 50 independent simulation runs. Here we study the scaling behaviour of the normalized and circular averaged SSF, $I(k, t)$, defined as [9]

$$I(k, t) \equiv S(k, t) \Big/ \sum_k k^2 S(k, t) \tag{7}$$

with

$$S(k, t) \equiv \frac{1}{N^2} \left[\left\langle \left| \sum_{\boldsymbol{n}} A(\boldsymbol{n}, t)\, e^{i\boldsymbol{k}\cdot\boldsymbol{n}} \right|^2 \right\rangle + \left\langle \left| \sum_{\boldsymbol{n}} B(\boldsymbol{n}, t)\, e^{i\boldsymbol{k}\cdot\boldsymbol{n}} \right|^2 \right\rangle \right] \tag{8}$$

where the bracket $\langle \ldots \rangle$ denotes the circular average in k space for $k = |\boldsymbol{k}|$ [9]. In figures 3 and 4 we show the scaled SSF, $\bar{k}^2 I(k, t)$, plotted against the reduced wave number $k/\bar{k}$ at various times for the conserved and the non-conserved systems, respectively. Here the average wave number $\bar{k}$ is defined by

$$\bar{k} = \sum k I(k, t) \Big/ \sum I(k, t) \tag{9}$$

where the summations are taken over $0 < k < \pi$ [9]. The time evolutions of the characteristic length scale, $\bar{k}^{-1}$, are plotted for both systems in figure 5. In our simulations we

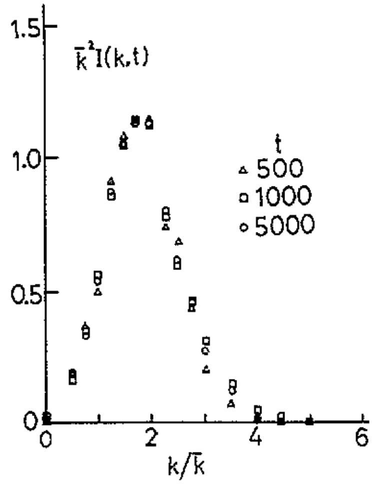

Figure 3. Scaled scattering structure factor $\bar{k}^2 I(k,t)$ versus $k/\bar{k}$ for a conserved system at times $t = 500$ (△), 1000 (□) and 5000 (○).

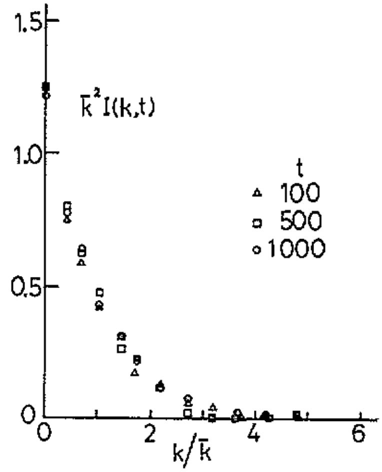

Figure 4. Scaled scattering structure factor $\bar{k}^2 I(k,t)$ versus $k/\bar{k}$ for a non-conserved system at times $t =$ 100 (△), 500 (□) and 10^3 (○).

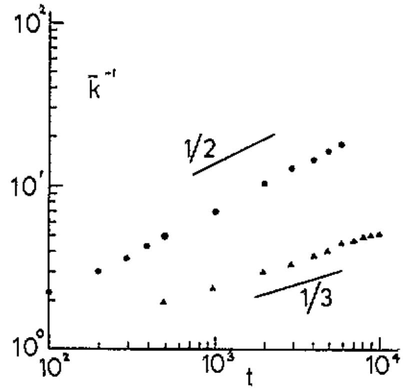

Figure 5. Time evolution of the characteristic length scales $\bar{k}^{-1}$ for a conserved (▲) and a non-conserved (●) systems. Straight lines are also shown with slopes indicated.

found that the characteristic length scale grows in time t as t^z with $z = 0.32 \pm 0.02$ for the conserved system and $z = 0.51 \pm 0.03$ for the non-conserved one, and for both systems the SSF obeys the asymptotic scaling law. The above values of the growth exponent are common to those of many models for quenched non-degenerate systems for the conserved and the non-conserved order parameters, respectively. Moreover, we have checked that the scaling behaviour of SSF seems to be valid for $t > t_0$ (in the present simulations, $t_0 \simeq 400$ for the conserved order parameter and $t_0 \simeq 30$ for the non-conserved one), where t_0 is a time at which the volume fraction of ordered lattice sites becomes larger than 0.7.

In summary, we have numerically studied the dynamical behaviour after a quench in triply degenerate systems both with the conserved and the non-conserved order parameter. We have found that even the ordering process of quenched systems with triply degenerate ordered states obeys the asymptotic dynamical scaling law, similar to that of non-degenerate systems, with the growth exponent z of the characteristic length scale being about $1/3$ for the conserved system and about $1/2$ for the non-conserved one, respectively. In the present letter, we have, however, simulated only a $p = 3$ system. Further simulations are thus needed, changing the degree of degeneracy p and/or the parameter v. Moreover, the detailed forms of the scaled function of SSF and comparison of them with those of non-degenerate systems are interesting, as well as the degeneracy dependence of the scaled function and the growth exponent. These problems still remain open to us.

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